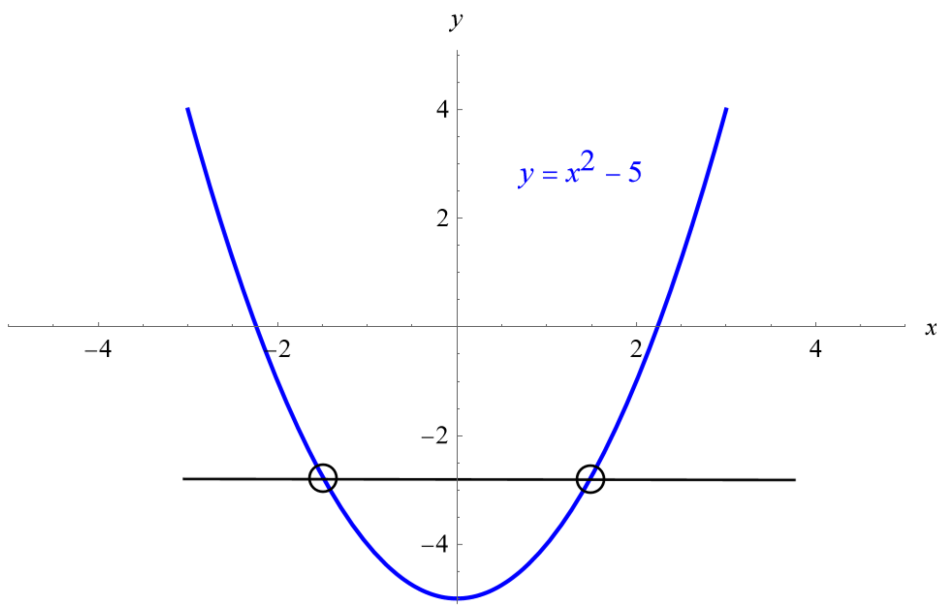
Exercise 15

For the following exercises, find a domain on which each function f is one-to-one and non-decreasing. Write the domain in interval notation. Then find the inverse of f restricted to that domain.

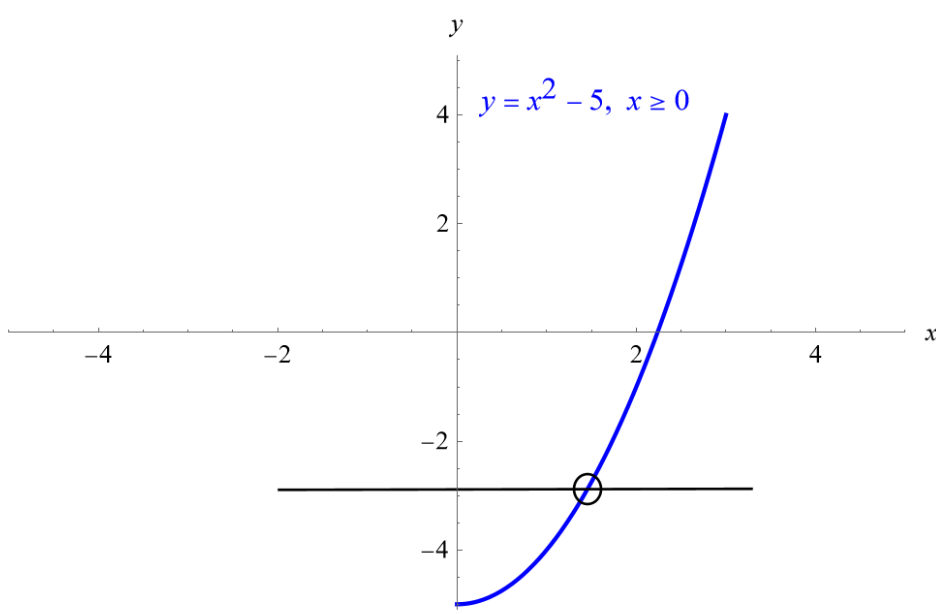
$$f(x) = x^2 - 5$$

Solution

This function is not one-to-one because it fails the horizontal line test.



But it can be made one-to-one by taking the restriction of $f(x)$ to $x \geq 0$.



The domain on which $f(x) = x^2 - 5$ is one-to-one and non-decreasing is $[0, \infty)$. To find the inverse, switch x and y.

$$x = y^2 - 5$$

Solve for y.

$$y^2 = x + 5$$

Take the square root of both sides.

$$\sqrt{y^2} = \sqrt{x+5}$$

Since there's an even power under an even root and the result is odd, an absolute value sign is needed.

$$\sqrt{y} = \sqrt{x+5}$$

Remove the absolute value sign by placing ± on the left side.

$$y = \pm\sqrt{x+5}$$

In order to decide whether to choose the plus or minus sign, notice that y originally came from x, which has the domain $[0, \infty)$. Choosing the minus sign would allow values of y less than 0. Therefore, the inverse function is

$$f^{-1}(x) = \sqrt{x+5}.$$